

# Ladywood Primary School



## Calculation Policy September 2020

Ladywood

Primary School

## **Maths Calculation Policy**

At Ladywood we have based our maths curriculum on the White Rose Maths scheme of work. This ensures that our children are able to progress consistently across school. This policy reflects this throughout. The progression is in line with the Programme of Study in the 2014 National Curriculum.

This calculation policy should be used to support children to develop a deeper understanding of number and calculation. Differentiation should primarily be through support, scaffolding and deepening, not through task. This ensures that children develop their mathematical understanding at the same pace, and therefore master the objectives taught. It may be necessary for children to work in mixed ability groups at times. Evidence shows that this increases the confidence of the less able children, impacting positively upon outcomes. We must also ensure that the learning of the most able pupils is being extended. Children should access greater depth challenges in most lessons but we must be careful that these are not just an extension but that they are activities that deepen their thinking and understanding. We recommend the mastery challenges on the NCTEM website which is supported by the South Yorkshire Maths Hub.

This policy is designed to support our children to develop a greater understanding of number through the use of concrete, pictorial and abstract representations.

These principles (concrete, pictorial and abstract) should not be confused with differentiation for lower, middle and higher attaining. This approach is an approach to be used with the whole class, with each principle being equally important. Manipulatives should not be seen as a resource just to support the lower attaining children or the less confident.

**Concrete representation-** a pupil is first introduced to an idea or skill by acting it out with real objects. This is a 'hands on' component using real objects and is a foundation for conceptual understanding.


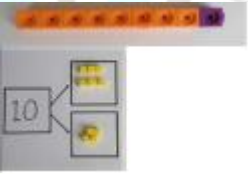

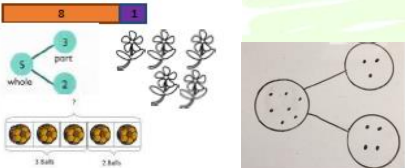
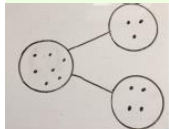
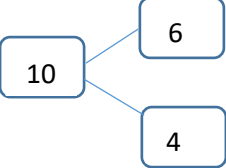
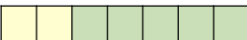
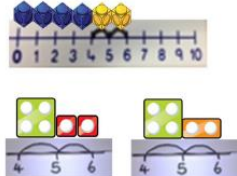
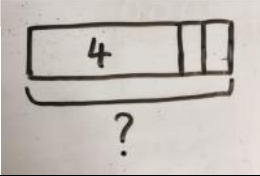

**Pictorial representation** – a pupil has sufficiently understood the 'hands on' experiences performed and can now relate them to representations, such as a diagram or picture of the problem.

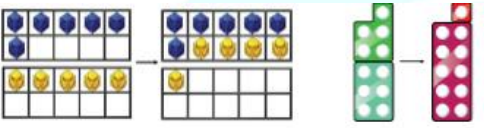
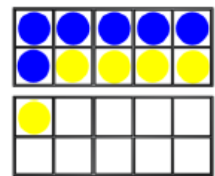

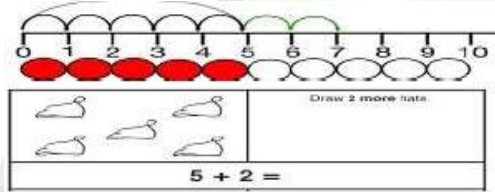
**Abstract representation**—a pupil is now capable of representing problems by using mathematical notation, for example  $12 \times 2 = 24$ .

# ADDITION

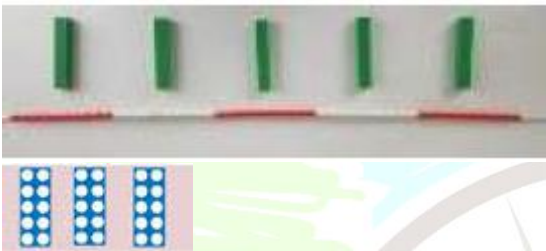

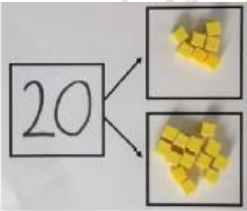
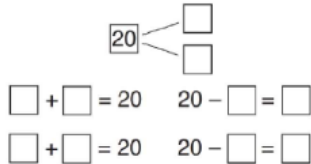
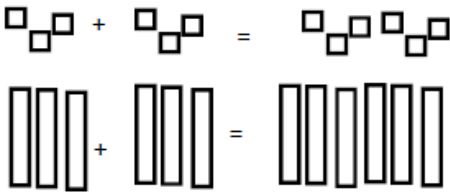
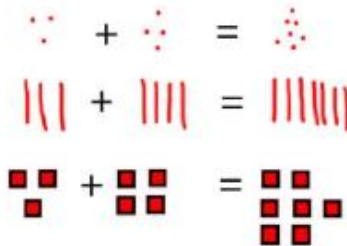
Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

## YEAR 1 ADDITION

Objective/ Strategy	Concrete	Pictorial	Abstract
<b>Combining two parts to make a whole: part-whole model.</b> <b>Joining two groups and then recounting all objects. (lots of practise making 10 and numbers to 10)</b>	$4 + 3 = 7$  Use part, part whole model. Use cubes to add two numbers together in a group or in a bar.  Here are seven pieces of fruit.  Put the fruit into a part-whole model. Complete the sentences. _____ is the whole. _____ is a part, _____ is a part and _____ is a part.	Use pictures to add two numbers together as a group or a bar.  	$10 = 4 + 6$ $4 + 6 = 10$  Include missing number calculations. $? = 4 + 6$ $4 + ? = 10$ Complete the number sentences.  $\_ + \_ = 7$ $7 = \_ + \_$ $\_ + \_ = 7$ $7 = \_ + \_$
<b>Start at the bigger number and count on.</b>	Start with the larger number of equipment and count on to the smaller number 1 by 1 to find the answer. 	Use a bar model that encourages children to count on rather than count them all. So 4, 5, 6 instead of 1, 2, 3, 4, 5, 6. 	An abstract number line- in your head. Put the larger number in your head and count on the smaller number until you find your answer. 

<p><b>Regrouping to make 10</b>  <i>An essential skill for column addition later</i></p>	<p>Using tens frames and counters or Numicom.  <math>6 + 5 = 11</math>            First there are 6            Then there are 5 more            Now there are 11</p> 	<p>Children to draw the counters on a tens frame.</p> 	<p>Children to develop an understanding of equality.</p> <p><math>6 + ? = 11</math></p> <p><math>6 + 5 = 5 + ?</math></p> <p><math>6 + 5 = ? + 5</math></p>
<p><b>Represent and use number bonds and related subtraction facts within 20.</b></p>	<p>2 more than 5 is...</p> 		<p>Include missing number operations.            Emphasise the language.</p> <p>5 and 2 is 7            7 is 2 more than 5            5 is 2 less than 7</p>

## YEAR 2 ADDITION

Objective/ Strategy	Concrete	Pictorial	Abstract
<b>Adding multiples of 10</b>	<p>Model using Base 10, Numicon, bead strings etc.</p> 	<p>Use Base 10 representations.</p> 	$20 + 30 = 50$ $50 + 20 = 70$ $40 + ? = 50$
<b>Use known number facts</b> Part, part whole	<p>Children to explore ways of making numbers within 20.</p> 	 $\begin{array}{l} 20 \swarrow \searrow \\ \square + \square = 20 \quad 20 - \square = \square \\ \square + \square = 20 \quad 20 - \square = \square \end{array}$	<p>Explore commutativity of addition by swapping the addends to build a fact family.</p> <p>Explore the concept of the inverse relationship of addition and subtractions and use this to check calculations.</p> $\begin{array}{ll} \square + 1 = 16 & 16 - 1 = \square \\ 1 + \square = 16 & 16 - \square = 1 \end{array}$
<b>Using know facts</b>		<p>Children to draw representations of Base 10.</p> 	<p>Make connections so;</p> $3 + 4 =$ leads $30 + 40 = 70$ leads to $300 + 400 = 700$

**Bar model**

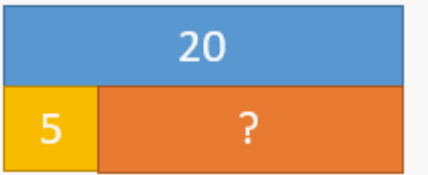
$3 + 4 = 7$



$7 + 3 = 10$

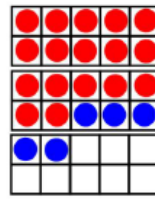


$6 + 2 = 8$



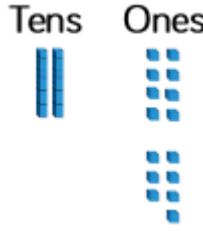
$20 - ? = 5$

**Add a two digit number and ones**



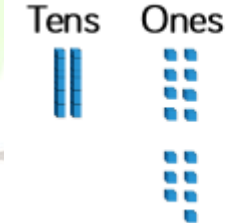
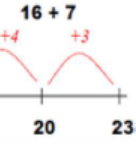
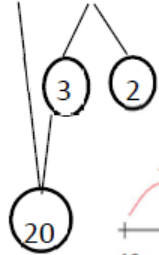
$17 + 5 = 22$

Use the tens frame to make a 'complete ten'



Use part, part whole and number line to model.

$17 + 5 = 22$



$$\begin{array}{r} 28 \\ + 7 \\ \hline 35 \\ 1 \end{array}$$

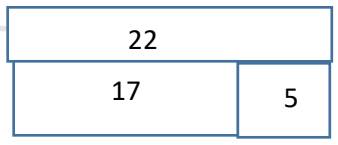
Explore related facts

$17 + 5 = 22$

$5 + 17 = 22$

$22 - 17 = 5$

$22 - 5 = 17$



$$\begin{array}{r} 28 \\ + 7 \\ \hline 35 \\ 1 \end{array}$$

**Add a two digit number and tens**


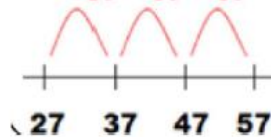
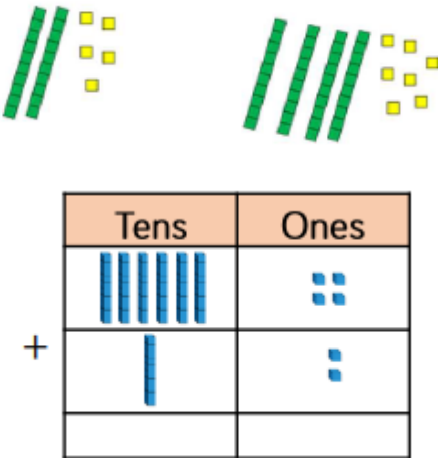
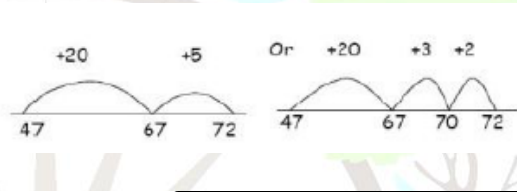

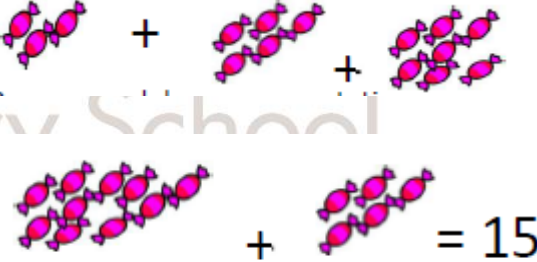
Explore the ones that do not change.

Use a number line to show this.

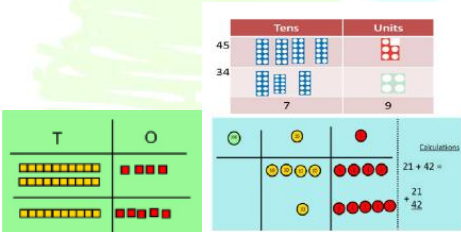

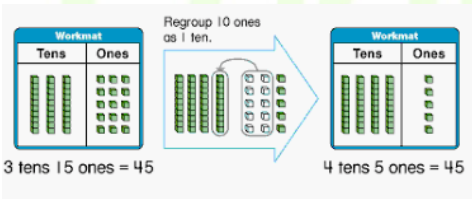
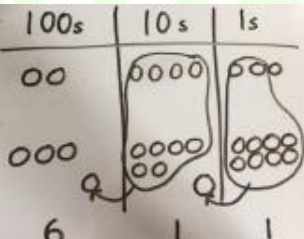
Record as calculations.

$27 + 10 = 37$

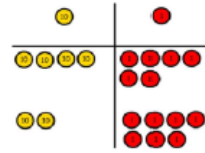


		<p><b>27 + 30</b></p> <p>+10 +10 +10</p>  <p>27 37 47 57</p>	<p>27 + 20 = 47</p> <p>27 + ? = 57</p>										
<b>Add a two 2- digit numbers</b>	<p>Model using Base 10, place value counters and Numicon.</p>  <table border="1" data-bbox="553 697 900 965"><thead><tr><th>Tens</th><th>Ones</th></tr></thead><tbody><tr><td></td><td></td></tr><tr><td>+</td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></tbody></table>	Tens	Ones			+						<p>Use a number line and bridge ten using part, part whole if needed.</p>  <p>47 67 72      Or      47 67 70 72</p> <p>64 + 12 = ____</p> <p>4 ones + 2 ones = ____</p> <p>6 tens + 1 ten = ____</p> <p>____ tens + ____ ones = ____</p>	<p>25 + 47</p> <p>20 + 5      40 + 7</p> <p>20 + 40 = 60</p> <p>5 + 7 = 12</p> <p>60 + 12 = 72</p> <p>64 + 12 ____ ____</p>
Tens	Ones												
+													
<b>Add three 1 digit numbers</b>	<p>Combine to make 10 first or bridge 10.</p> 	<p>Regroup and draw representation.</p>  <p>4 + 7 + 6 = 15</p>	<p>Combine the two numbers that make/ bridge ten then add on the third number.</p> <p>4 + 7 + 6 = 10 + 7 = 17</p>										

## YEAR 3 ADDITION

Objective/ Strategy	Concrete	Pictorial	Abstract
<b>Column Addition- no regrouping</b>  <b>Add two or three 2 or 3 digit numbers</b>	<p>Use Base 10, Numicon, place value counters to represent the numbers.</p> 	<p>Draw the tens and ones</p> 	<p>Add the ones first, then add the tens and then add the hundreds.</p> $\begin{array}{r} 354 \\ + 224 \\ \hline \end{array}$
<b>Column addition with regrouping</b>	<p>Exchange ten ones for a ten. Model this using Numicon, Base 10 and place values counters. <b><i>Allow the children to practise this step to ensure understanding.</i></b></p> 	<p>Draw representations to further support their understanding. Children to circle when they make an exchange.</p> 	<p>Start by partitioning the numbers before column addition. This shows the exchange. e.g.</p> $\begin{array}{l} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$





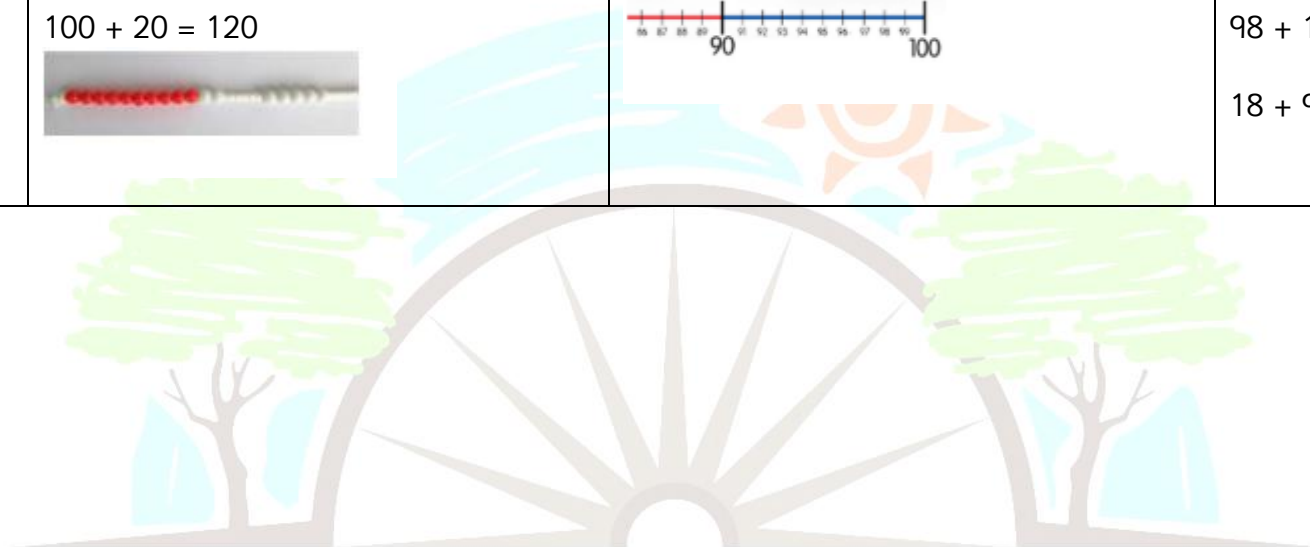


$$46 + 27 = 73$$

# Ladywood








































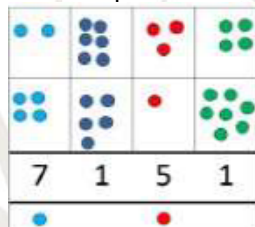
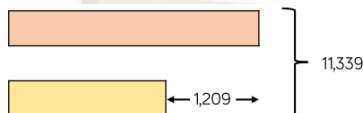
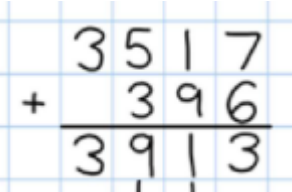



















## Primary School

<p><b>Estimate the answers to questions and use the inverse operations to check answers</b></p>	<p>Estimate the answers to questions by using equipment to support. Bead strings are good for this.</p> $98 + 17 =$ $100 + 20 = 120$ 	<p>Use a number line to illustrate an estimation.</p> 	<p>Build up known facts and use these to illustrate the inverse and to check the answers.</p> $98 + 18 = 116 \quad 116 - 18 = 98$ $18 + 98 = 116 \quad 116 - 98 = 18$
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Ladywood  
Primary School

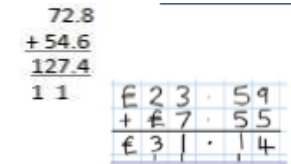
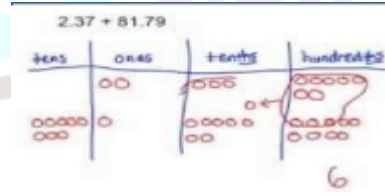
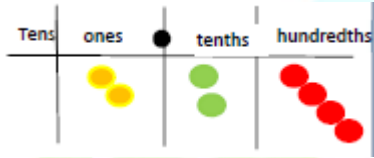
# YEARS 4 - 6 ADDITION

Objective/ Strategy	Concrete	Pictorial	Abstract												
<b>Y4 – 6</b> <b>Estimate the answers to questions and use the inverse operations to check answers</b>	AS YEAR 3														
YEAR 4															
<b>Add numbers with 3 digits</b>	<p>Children continue to use Base 10 and place value counters to show an exchange.</p>  <p>Use counters and a place value grid to calculate <math>3,242 + 2,213</math></p> <table border="1"><thead><tr><th>1,000s</th><th>100s</th><th>10s</th><th>1s</th></tr></thead><tbody><tr><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td></tr></tbody></table>	1,000s	100s	10s	1s	  	 	   	 	 	 		  	<p>Draw representations using place value grid</p>  <p>Use bar model</p> 	<p>Continue with work on carrying tens to carrying 100s and 1000s. Try to relate to money, measures to keep maths real and relatable.</p> 
1,000s	100s	10s	1s												
  	 	   	 												
 	 		  												

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Primary School

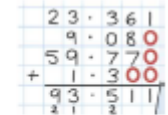
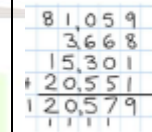
## YEAR 5

<b>Add numbers with more than 4 digits</b>	As year 4		
<b>Add decimals with 2 decimal places, including money.</b>	Introduce decimal place value counters and model exchange for addition.	Draw the representations in the place value counters.	Practise using numbers then as money to support their understanding of decimals.




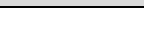









## YEAR 6

<b>Add several numbers of increasing complexity, including adding money, measure and decimals with different numbers of decimal points.</b>	As year 5, model using place value counters.	Children to be encouraged to draw their representations.	Insert zeros for place holders.
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It is important that we ask children questions in a variety of ways. (Conceptual variation)

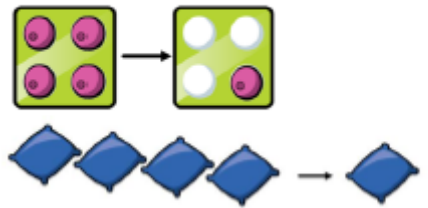
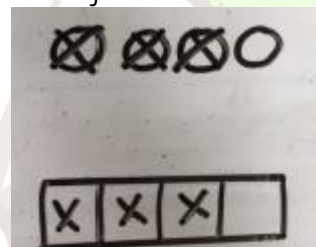
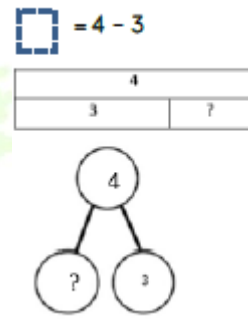
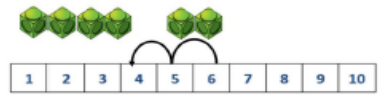

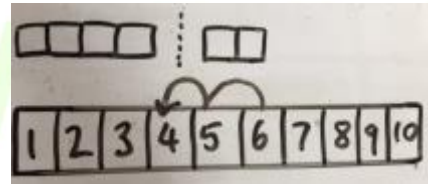
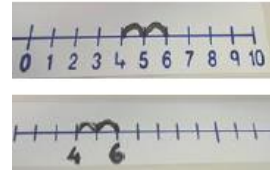
### Conceptual Variation- Different ways to solve $21 + 34$

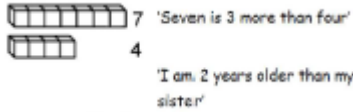

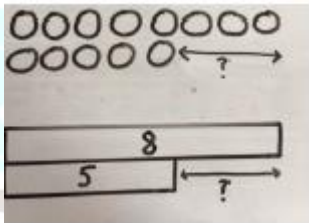
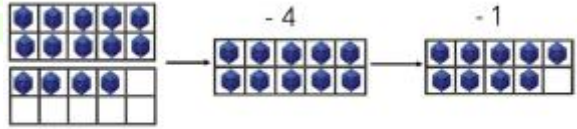

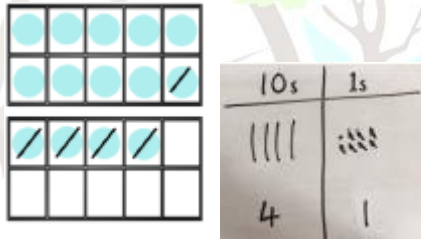
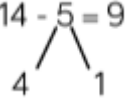

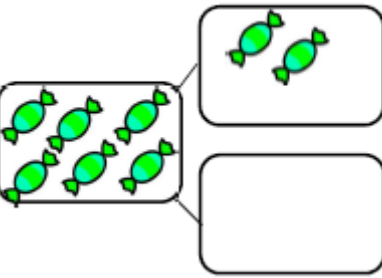

 <table border="1" data-bbox="109 1224 315 1236"><tr><td colspan="2">?</td></tr><tr><td>21</td><td>34</td></tr></table>	?		21	34	<p>Word problems.</p> <p>In year 3 there are 21 children and in year 4 there are 34 children. How many children in total?</p> <p>21 + 34 = 55- Prove it</p>	<p>21 + 34 =</p> <p>? - 21 = 34 Inverse</p> <p>Calculate the sum of twenty one and thirty four.</p>	 <p>Missing digit problems:</p> <table border="1" data-bbox="1776 1195 1942 1224"><thead><tr><th>10s</th><th>1s</th></tr></thead><tbody><tr><td></td><td></td></tr><tr><td></td><td>?</td></tr><tr><td>?</td><td>5</td></tr></tbody></table>	10s	1s				?	?	5
?															
21	34														
10s	1s														
															
	?														
?	5														

# SUBTRACTION

Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

## YEAR 1 SUBTRACTION

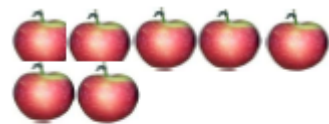
Objective/ Strategy	Concrete	Pictorial	Abstract
<b>Taking away ones</b>	<p>Children to use physical objects to 'take away'</p> <p><math>4 - 3 = 1</math></p> 	<p>Children to draw the objects that they are using and cross out the ones that they 'take away' They can also use a bar model for this.</p> 	<p><math>4 - 3 =</math></p> <p></p>
<b>Counting back</b>	<p>Model moving objects away from the group as you count back. Children to practise this.</p> <p><math>6 - 2 = 4</math></p>  <p>Move beads along the bead string as they count back.</p> 	<p>Children to represent what they see pictorially.</p> 	<p>Put the starting number in your head and count back the number you are taking away</p> <p><math>6 - 4 =</math></p>  <p>6 in your head count back 2. 6, 5, 4</p>

<b>Finding the difference</b>	<p>Compare objects and amounts</p>  <p>Lay objects to represent the bar model.</p> 	<p>Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.</p> 	<p>Find the difference between 8 and 5.  <math>8 - 5</math>, the difference is ____</p> <p>Children to explore why  <math>9 - 6 =</math>  <math>8 - 5 =</math>  <math>7 - 4 =</math> have the same difference.</p>
<b>Make 10</b>	<p>Make 10 using tens frames, Numicon, bead strings etc.</p>  	<p>Children to present the ten frame pictorially and discuss what they did to make 10.</p> 	<p>Children to show how they can make 10 by partitioning the subtrahend.  <math>14 - 4 = 10</math>  <math>10 - 1 = 9</math></p> <p><math>14 - 5 = 9</math></p> 
<b>Represent and use number bonds and related subtraction facts within 20</b> <b>Include subtracting zero</b> <b>Part part Whole model</b>	<p>Link to addition use PPW model to model to inverse.</p>  <p>So if 10 is the whole and 6 is one of the parts,  What's the other part?  <math>10 - 6 = 4</math></p>	<p>Use pictorial representations to show the part.</p> 	<p>Move to using numbers within the part whole model.</p>  <p>Include missing number problems:  <math>12 - ? = 5</math>  <math>7 = 12 - ?</math></p>

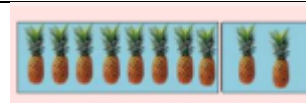


**Bar model**  
**Including the**  
**Inverse operations.**

Children to set out the objects as a bar model.



$$5 - 2 = 3$$



8

2

$$10 = 8 + 2$$

$$10 = 2 + 8$$

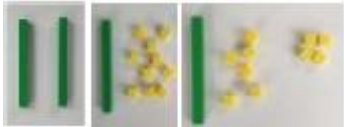

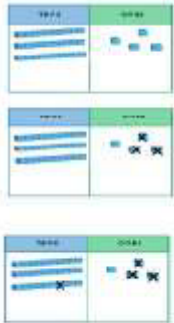
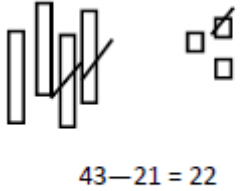
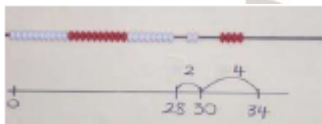
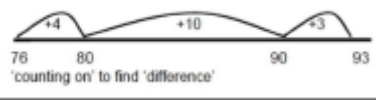
$$10 - 2 = 8$$

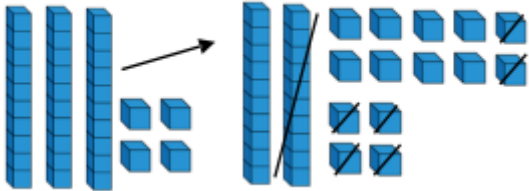
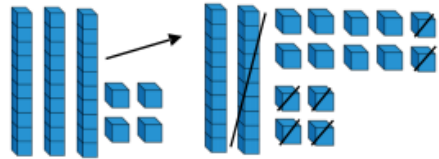
$$10 - 8 = 2$$

# Ladywood





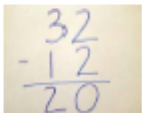
## Primary School

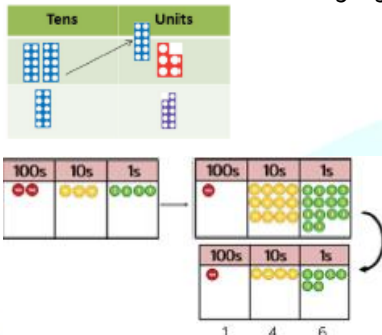
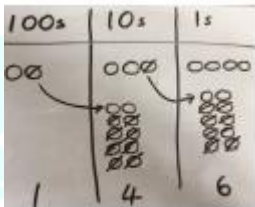
## YEAR 2 SUBTRACTION

Objective/ Strategy	Concrete	Pictorial	Abstract
<b>Regroup a ten into 10 ones</b>	Use Base 10 or place value cards to represent 10 ones being regrouped into 1 ten. 	Children to cross out the numbers that they are taking away from the ten. 	Written $20 - 4 = 16$
<b>Partitioning to subtract without regrouping.</b>	Use Base 10 to show partitioning a number without regrouping. $34 - 13 =$ 	Children to draw their representation. 	$43 - 21 = 22$
<b>Make ten strategy</b> <b>Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.</b>	Make 10 first $34 - 28 =$ 	Use a number line to show. 	Written $34 - 28 =$

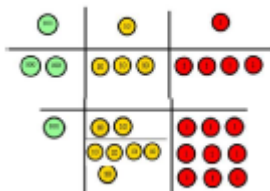
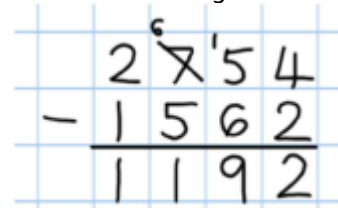
	<p>Take 16 away from 34</p> 	<p>Draw</p> <p>Take 16 away from 34</p> 	$\begin{array}{r} 2\cancel{3}14 \\ -16 \\ \hline 18 \end{array}$
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## YEAR 3 SUBTRACTION

Objective/ Strategy	Concrete	Pictorial	Abstract
<b>Subtract numbers mentally, including:</b> three digit number + ones three digit number + tens three digit number + hundreds	<p>Use resources to show</p> 	<p>Draw your working on a number line.</p> 	<p>Vary the position of the answer and include missing number calculation for the children to solve.</p> <p>*They may have to go back to concrete or pictorial to support this to begin with.</p>
<b>Column subtraction without regrouping (friendly numbers)</b>	 <p>47—32</p> <p>Use base 10 or Numicon to model</p>	 <p>Draw representations to support understanding</p> <p>Calculations</p> $\begin{array}{r} 54 \\ -22 \\ \hline 32 \end{array}$	$47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ -20 + 4 \\ \hline 20 + 3 \end{array}$ <p>Intermediate step may be needed to lead to clear subtraction understanding.</p> 

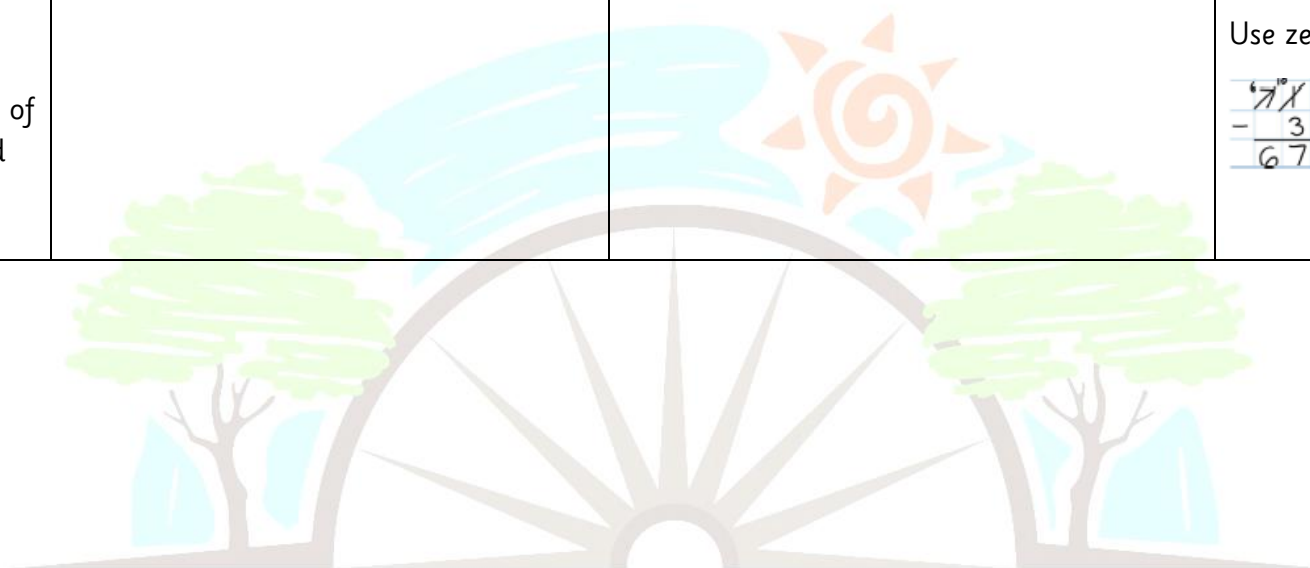
<b>Column subtraction with regrouping</b>	<p>Begin with Numicon. Move to place value counters to model exchanging tens and ones.</p> 	<p>Children may draw PV counters or Base 10 and cross off to show the exchange.</p> 	<p>Formal method. Children <b>MUST</b> understand what they are doing when they are exchanging. Concrete and pictorial steps <b>VERY</b> important.</p> $\begin{array}{r} 234 \\ - 88 \\ \hline 6 \end{array}$
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## YEARS 4 - 6 SUBTRACTION

Objective/ Strategy	Concrete	Pictorial	Abstract
<p><b>Y4 – 6</b></p> <p><b>Subtracting tens and ones.</b></p> <p><b>Year 4 subtract up to 4 digits.</b></p>	<p>Model the process of exchanging using PV counters.</p> <p>234 - 179</p> 	<p>Children to draw visual representation of the exchange using PV counters as per Y3.</p>	<p>Children MUST understand what they are doing. Not just crossing out and putting the carrier in the next column.</p> <p>Allow the children access to PV counters to support this/ ensure they 'draw' at least one to show you their understanding.</p> 

## YEAR 5

<p><b>Subtract with at least 4 digits, including money and measures.</b></p> <p>Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal</p> <p>Up to 3 decimal places</p>	<p>As Year 4- use PV counters to physically exchange.</p>	<p>As Year 4- draw PV counters to show the physical exchange.</p>	<div data-bbox="1747 135 1948 247"> <math display="block">\begin{array}{r} 28'10'8'' \\ - 2'12'' \\ \hline 28'9'2'' \end{array}</math> </div> <p>Use zero as a place holder.</p> <div data-bbox="1657 327 1870 438"> <math display="block">\begin{array}{r} 78'68''0 \\ - 372'5 \\ \hline 6796'5 \end{array}</math> </div>
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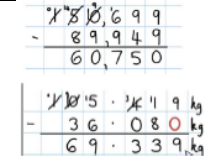
Ladywood  
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## YEAR 6

**Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal place).**

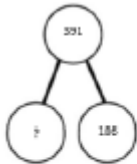
As Year 4- use PV counters to physically exchange.

As Year 4- draw PV counters to show the physical exchange.



It is important that we ask children questions in a variety of ways. (Conceptual variation)

### Conceptual Variation- Different ways to solve 391- 186



391	
186	?

Raj spent £391, Timmy spent £186.  
How much more did Raj spend?

Calculate the difference between 391 and 186.

$$\square = 391 - 186$$

$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

What is 186 less than 391?

Missing digit calculations

$$\begin{array}{r} 39\square \\ -\square\square6 \\ \hline \square05 \end{array}$$

Primary School

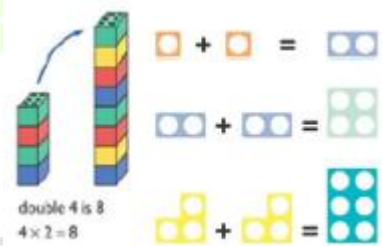

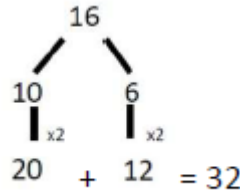

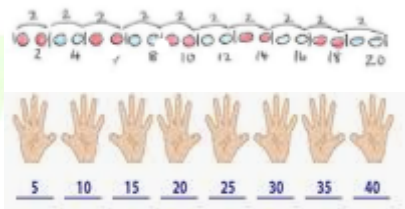


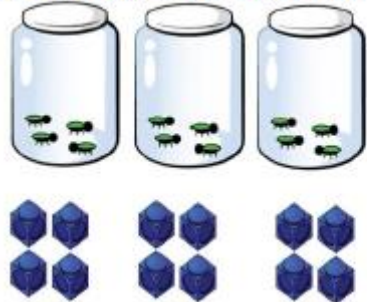
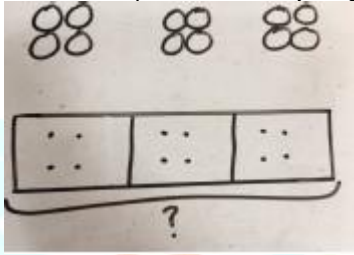
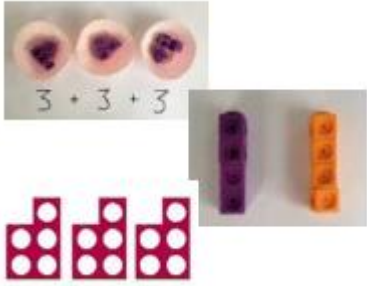
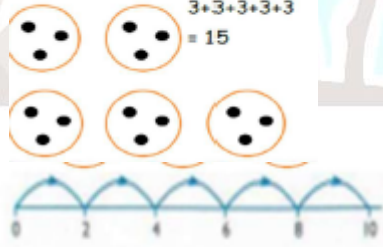
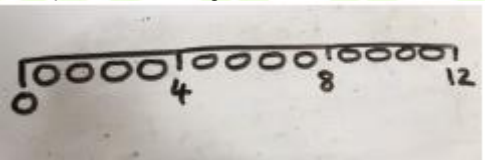




# MULTIPLICATION

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

## YEAR 1 MULTIPLICATION

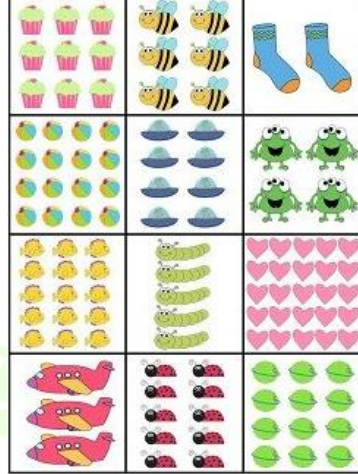
*Programme of Study specifies the following objectives, however it does not require the explicit teaching of the mathematical symbol of multiplication*

Objective/ Strategy	Concrete	Pictorial	Abstract
<b>Doubling</b>	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling</p> 	<p>Draw pictures to show how to double numbers</p> 	<p>Partition a number and then double each part before recombining it back together.</p> 
<b>Counting in multiples (2, 5, 20)</b>	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting.</p> 	<p>Children make representations to show counting in multiples.</p> 	<p>Count in multiples of a number aloud. Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, , 10...</p> <p>5, 10, 15, 20, 25...</p> <p>10, 20, 30, 40, 50 ...</p> <p>Remember to practise counting backwards and from different starting points, including missing numbers.</p>

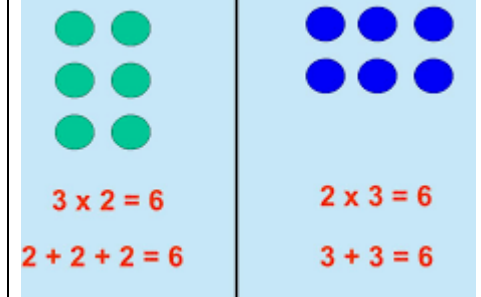
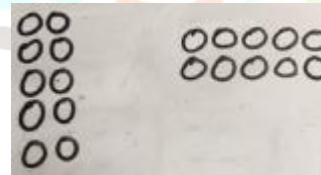
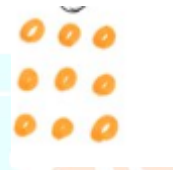
<p><b>Making equal groups and counting the total</b></p>	<p>Making equal groups with objects. There are 3 equal groups, with 4 in each group.</p> 	<p>Draw to represent the equal groups.</p> 	<p><math>3 \times 4 = 12</math></p>
<p><b>Repeated addition</b></p>	<p>Use a variety of manipulatives to add equal groups.</p> 	<p>Use pictures, including number lines to show the repeated addition.</p> <p>There are 3 sweets in one bag. How many sweets are in 5 bags altogether?</p> <p><math>3 + 3 + 3 + 3 + 3 = 15</math></p>  <p>Use pictorial alongside a number line.</p> 	<p>Write addition sentences to describe objects and pictures.</p> <p><math>2 + 2 =</math> </p> <p><math>2 + 2 + 2 =</math> </p> <p><math>2 + 2 + 2 + 2 =</math> </p> <p><math>2 + 2 + 2 + 2 =</math> </p>

## Understanding arrays

Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.



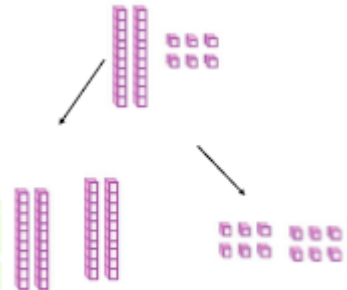

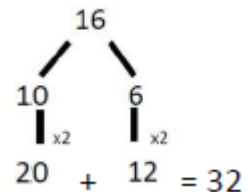






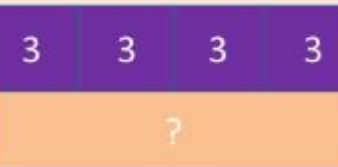
Children to draw to represent the arrays.

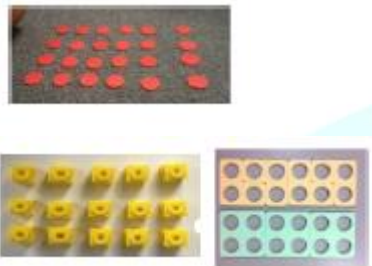

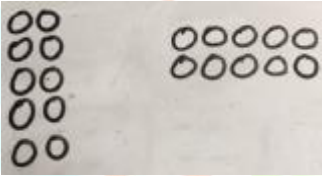


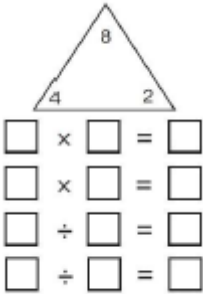


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## YEAR 2 MULTIPLICATION

Children should be able to recall and use multiplication and division facts for the 2, 5 and 10 times tables.

Objective/ Strategy	Concrete	Pictorial	Abstract
<b>Doubling</b>	<p>Model doubling using dienes and PV counters.</p>  $40 + 12 = 52$	<p>Draw pictures and representations to show how to double numbers</p> 	<p>Partition a number and then double each part before recombining it back together.</p>  $\begin{array}{r} 16 \\ \swarrow \quad \searrow \\ 10 \quad 6 \\ \downarrow \times 2 \quad \downarrow \times 2 \\ 20 \quad + \quad 12 \\ \hline 32 \end{array}$
<b>Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)</b>	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.</p> <p><math>5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40</math></p>   <p><math>5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40</math></p> 	<p>Number lines, counting sticks and bar models should be used to show representation of counting in multiples.</p>    	<p>Count in multiples of a number aloud. Write sequences with multiples of numbers.</p> <p>0, 2, 4, 6, 8, 10  0, 3, 6, 9, 12, 15  0, 5, 10, 15, 20, 25, 30</p> <p><math>5 \times 5 = ?</math></p>

<p><b>Multiplication is commutative</b></p>	<p>Create arrays using counters and cubes and Numicon.</p>  <p>Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.</p> 	<p>Use representations of arrays to show different calculations and explore commutativity.</p> 	<p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$
<p><b>Using the Inverse This should be taught alongside division, so pupils learn how they work alongside each other.</b></p>		<p>Fact families</p> 	<p>Write/ show all related calculations.</p> $2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$



# YEAR 3 MULTIPLICATION

Children should be able to recall and use multiplication facts for the 3,4, and 8 times tables

## Objective/ Strategy

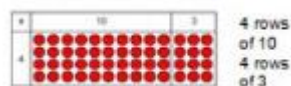
**Grid method, progressing to the formal method**

**Multiply 2 digit numbers by 1 digit numbers**

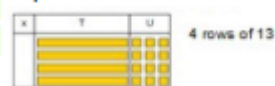
**Solve problems, including missing number problems, integer scaling problems.**

## Concrete

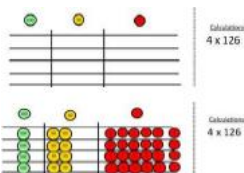
Show the links with arrays to first introduce the grid method.



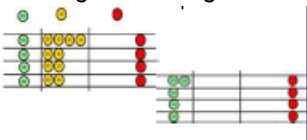
Move onto base ten to move towards a more compact method.



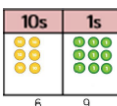
Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows. Fill each row with 126.



Add up each column, starting with the ones making any exchanges needed. Then you have your answer.

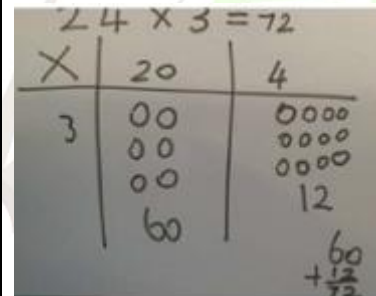


Formal column method with place value counters (base 10 can also be used)  $3 \times 23$

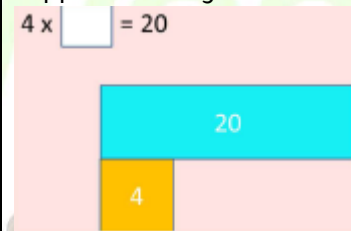


## Pictorial

Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.



Bar models can also be used to support missing numbers.



## Abstract

Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

$\times$	30	5
7	210	35

$$210 + 35 = 245$$

Move forward to the formal written method:

Children to record what it is they are doing to show understanding.

$$3 \times 23 = 69$$

$$3 \times 20 = 60$$

$$3 \times 3 = 9$$

$$60 + 9 = 69$$

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$



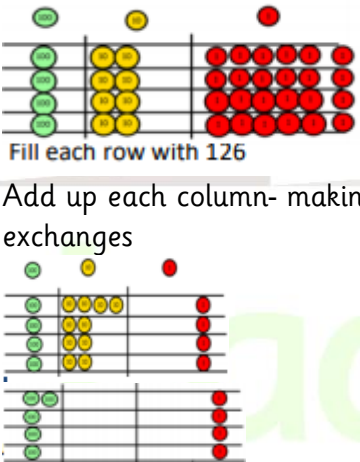
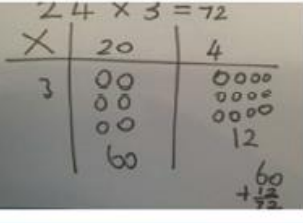
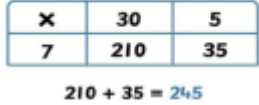

*When children start to multiply  $3d \times 3d$  and  $4d \times 2d$  etc., they should be confident with the abstract:*

*To get 744 children have solved  $6 \times 124$ .*

*To get 2480 they have solved  $20 \times 124$ .*

## YEAR 4 MULTIPLICATION


Children in year 4 should know times tables up to  $12 \times 12$  (Expectation)

Objective/ Strategy	Concrete	Pictorial	Abstract
<b>Grid method to recap from Year 3 for 2 digit by 1 digit</b>  <b>Multiply 3 digit numbers by 1 digit. (Year 4 expectation)</b>	<p>Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p>  <p>Calculations <math>4 \times 126</math></p>	<p>Children can represent their work with place value counters (draw) in a way that they understand.</p> 	<p>Start by multiplying by 1 digit number and represent on a grid.</p> 
<b>Column multiplication</b>	<p>Children can continue to use PV counters to support their understanding. Initially when there is no regrouping.</p> <p>It is important that, at this stage they always multiply the ones first</p> <p><math>327 \times 4 =</math></p>	<p>The grid method may be used to show how this related to the formal written method.</p> 	<p>Formal written method</p>

Hundreds	Tens	Ones

Bar modelling can also support learners to visualise and understand the calculation.

$$\begin{array}{r}
 327 \\
 \times 4 \\
 \hline
 28 \\
 80 \\
 1200 \\
 \hline
 1308
 \end{array}$$



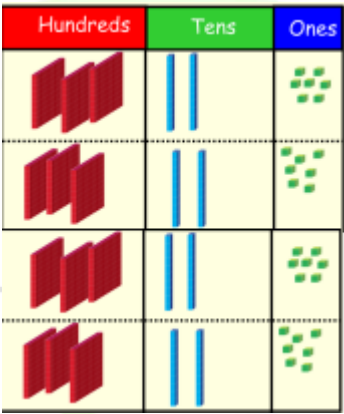
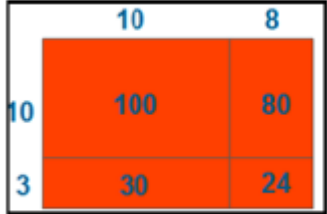
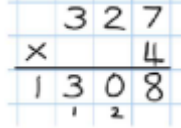
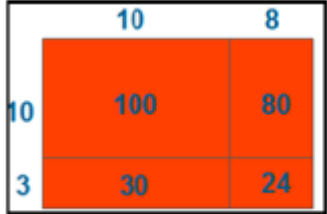
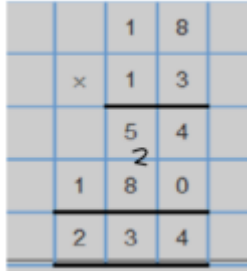
	3	2	7
x			4
	1	3	0
		2	8

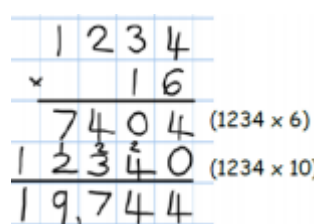
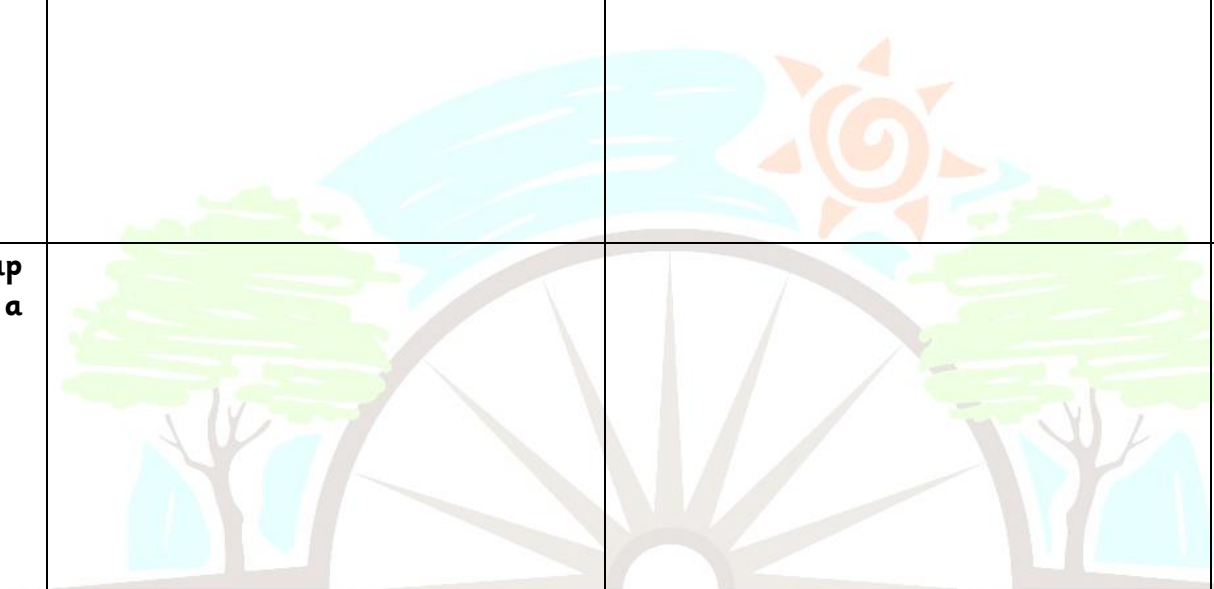
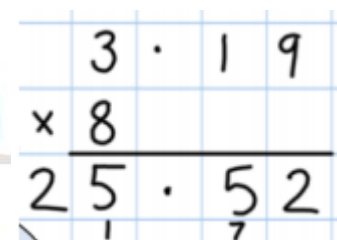
This may lead to a more compact method- Children MUST understand what they are 'doing' before moving onto this method.

Ask them to show you/ a peer/ talk about their working using drawings or PV counters.

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# YEAR 5/6

Objective/ Strategy	Concrete	Pictorial	Abstract								
<b>Column Multiplication for 3 and 4 digits x 1 digit.</b>	<p>Children can continue to use PV counters to support their understanding. Initially when there is no regrouping. It is important that, at this stage they always multiply the ones first <math>327 \times 4 =</math></p> 	<p>The grid method may be used to show how this related to the formal written method.</p> <table border="1" data-bbox="1081 395 1489 483"> <tr> <td>x</td><td>300</td><td>20</td><td>7</td></tr> <tr> <td>4</td><td>1200</td><td>80</td><td>28</td></tr> </table> <p>Bar modelling can also support learners to visualise and understand the calculation.</p> 	x	300	20	7	4	1200	80	28	<p>Formal written method</p> $\begin{array}{r} 327 \\ \times 4 \\ \hline 28 \\ 80 \\ \hline 1200 \\ \hline 1308 \end{array}$  <p>This may lead to a more compact method- Children MUST understand what they are 'doing' before moving onto this method.</p> <p>Ask them to show you/ a peer/ talk about their working using drawings or PV counters.</p>
x	300	20	7								
4	1200	80	28								
<b>Column multiplication</b>	<p>Manipulatives may still be used with the corresponding long multiplication modelled alongside.</p>	<p>Continue to use bar modelling to support problem solving</p> 	<p><math>18 \times 3</math> on the first row (<math>8 \times 3 = 24</math>, carrying the 2 for 20, then <math>1 \times 3</math>)  <math>18 \times 10</math> on the 2nd row. Show multiplying by 10 by putting zero in units first</p> 								



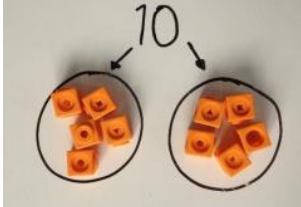

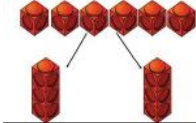

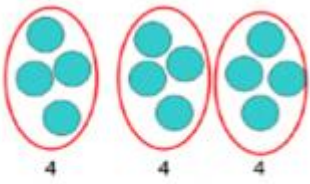
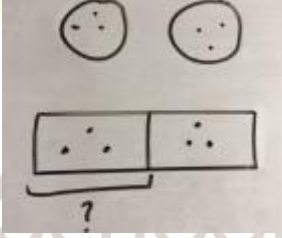
			
<b>Multiplying decimals up to 2 decimal places by a single digit.</b>		<p>Remind children that the single digit belongs in the ones column. Line up the decimal points in the question and the answer.</p> 	

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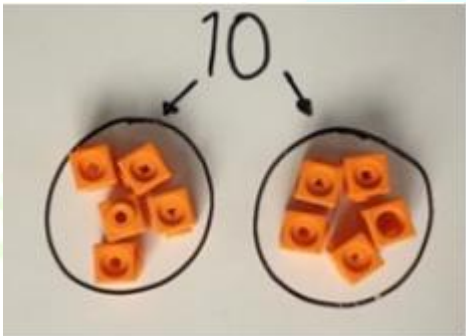
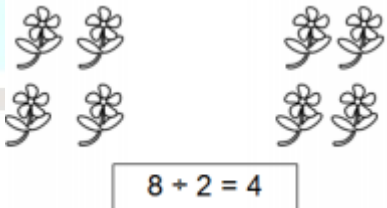
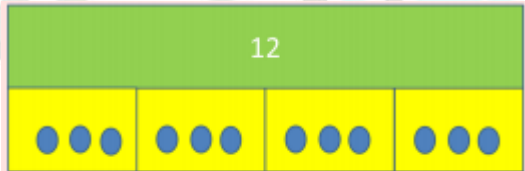
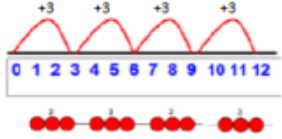
# DIVISION

Key language: share, group, divide, divided by, half.

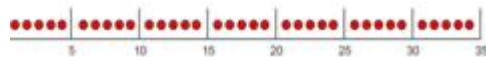
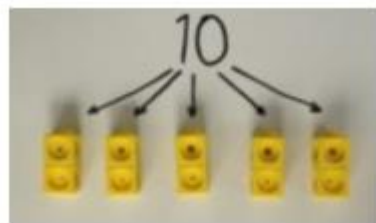
## YEAR 1 DIVISION

Objective/ Strategy	Concrete	Pictorial	Abstract
<b>Division as sharing</b>	<p>Share objects</p>   <p>I have 10 cubes/ sweets/ cakes etc. Can you share them equally between 2 groups/ children?</p>  <p>6 shared between 2</p>  	<p>Children to use pictures/ shapes to share quantities.</p>  <p>Sharing:</p>  <p>12 shared between 3 is 4</p> 	<p>10 shared between 2 is 5</p> <p>6 shared between 2 is 3</p> <p>12 shared between 3 is 4</p> <p>Children may be introduced to division symbol but wouldn't be expected to use it.</p>

## YEAR 2 DIVISION

Objective/ Strategy	Concrete	Pictorial	Abstract
<b>Division as sharing</b>	<p>I have 10 cubes can you share them equally between 2 groups?</p> 	<p>Children to use pictures or shapes to share quantities.</p>  <p>Children to use bar model to show and support understanding.</p>  <p><math>12 \div 4 = 3</math></p>	<p><math>12 \div 2 = 6</math></p> <p><math>10 \div 2 = 5</math></p> <p><math>12 \div 3 = 4</math></p>
<b>Division as grouping</b>	<p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p>	<p>Use number lines for grouping</p>  <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be</p>	<p><math>28 \div 7</math></p> <p>Divide 27 into 7 groups. How many in each group?</p>

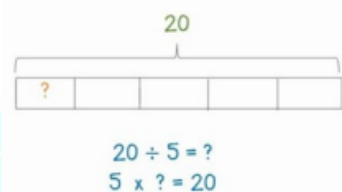




Use cubes, counters, objects or place value counters to aid understanding



within each group.



## Division with arrays

Link division to multiplication by creating an array and thinking about the number sentences that can be created.

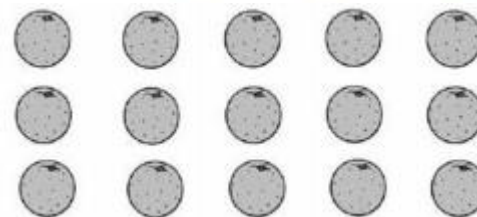
E.g.

$$15 \div 3 = 5 \quad 5 \times 3 = 15$$

$$15 \div 5 = 3 \quad 3 \times 5 = 15$$



Draw an array and use lines to split the array into groups to make multiplication and division sentences



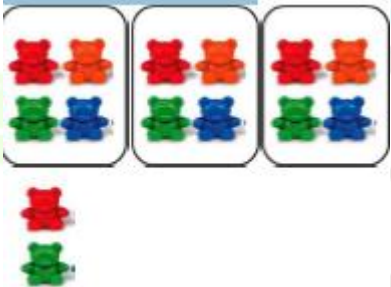


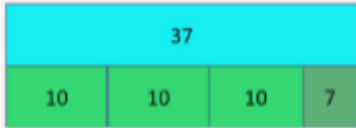
Find the inverse of multiplication and division sentences by creating eight linking number sentences.

$$7 \times 4 = 28 \quad 4 \times 7 = 28 \quad 28 \div 7 = 4 \quad 28 \div 4 = 7$$

$$7 = 28 \div 4 \quad 4 = 28 \div 7 \quad 28 = 7 \times 4 \quad 28 = 4 \times 7$$

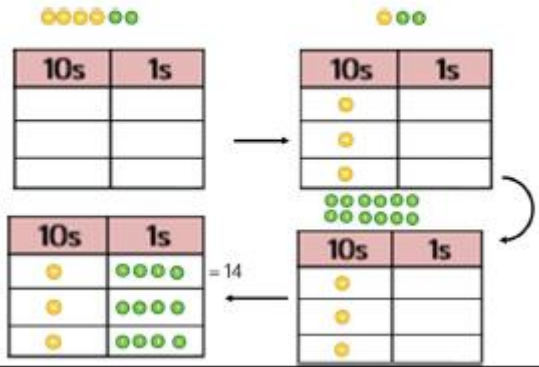
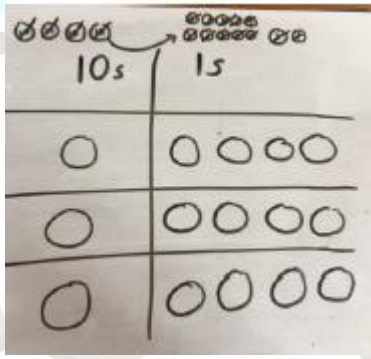
## YEAR 3 DIVISION

(Remember to look at Y2 C, P and A to support if needed before moving to remainders)

Objective/ Strategy	Concrete	Pictorial	Abstract
<b>Division with remainders</b>	<p><math>14 \div 3 =</math> Divide objects between groups and see how much is left over</p> 	<p>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.</p>  <p>Draw dots and group them to divide an amount and clearly show a remainder.</p>  <p>Use bar models to show division with remainders.</p> 	<p>Complete written divisions and show the remainder using r.</p> $29 \div 8 = 3 \text{ REMAINDER } 5$ <p>↑   ↑   ↑   ↑ dividend divisor quotient remainder</p>

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## YEARs 4 - 6 DIVISION

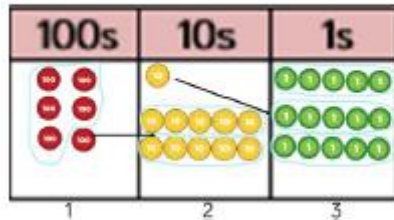
Objective/ Strategy	Concrete	Pictorial	Abstract
<b>Divide at least a 3 digit numbers by a 1 digit.</b>	<p>Sharing using place value counters.</p> $42 \div 3 = 14$ 	<p>Children to represent the place value counters pictorially.</p> 	<p>Children to be able to make sense of the place value counters and write calculations to show the process.</p> $42 \div 3$ $42 = 30 + 12$ $30 \div 3 = 10$ $12 \div 3 = 4$ $10 + 4 = 14$

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## Short Division

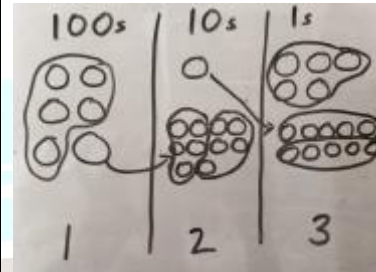
Use place value counters to divide using the bus stop method alongside

Short division using place value counters to group.  
 $615 \div 5$



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the PV counters.



Encourage the children to move towards counting in multiples to make their method more efficient.

Children to complete the calculation using the short division scaffold.

$$\begin{array}{r} 123 \\ 5 \overline{) 615} \end{array}$$

## Long division using place value counters

$$2544 \div 12$$

1000s	100s	10s	1s
●●	●●●●●●	●●●●	●●●●

We can't group 2 thousands into 12 so we will exchange them

1000s	100s	10s	1s
	●●●●●●●●	●●●●	●●●●

We can group 24 thousands into groups of 12 which leaves us with 1 hundred.

$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$$

1000s	100s	10s	1s
	●●●●●●●●	●●●●●●	●●●●

After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

1000s	100s	10s	1s
	●●●●●●●●		●●●●●●●●

After exchanging the 2 tens we have 24 ones. We can group 24 ones into 2 groups of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

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## Long Division

### Step 1 - A remainder in the ones

$$\begin{array}{r} \text{h t o} \\ 041 \text{ R}1 \\ 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0400 \text{ R}7 \\ 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ( $3,200 \div 8 = 400$ )

8 goes into 0 zero times (tens).

8 goes into 7 zero times, and leaves a remainder of 7.

$$\begin{array}{r} \text{h t o} \\ 061 \\ 4 \overline{) 247} \\ \underline{-4} \\ 3 \end{array}$$

When dividing the ones, 4 goes into 7 one time. Multiply  $1 \times 4 = 4$ , write that four under the 7, and subtract. This finds us the remainder of 3.

Check:  $4 \times 61 + 3 = 247$

$$\begin{array}{r} \text{th h t o} \\ 0402 \\ 4 \overline{) 1609} \\ \underline{-8} \\ 1 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply  $2 \times 4 = 8$ , write that eight under the 9, and subtract. This finds us the remainder of 1.

Check:  $4 \times 402 + 1 = 1,609$



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## Step 2- A remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ 2 \end{array}$ <p>Two goes into 5 two times, or 5 tens <math>\div 2 = 2</math> whole tens -- but there is a remainder!</p>	$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ -4 \\ \hline 1 \end{array}$ <p>To find it, multiply <math>2 \times 2 = 4</math>, write that 4 under the five, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ -4 \downarrow \\ \hline 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.</p>

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ -4 \\ \hline 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ -4 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>Multiply <math>9 \times 2 = 18</math>, write that 18 under the 18, and subtract.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ -4 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>The division is over since there are no more digits in the dividend. The quotient is 29.</p>

### Step 3 A remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \end{array}$ <p>Two goes into 2 one time, or 2 hundreds <math>\div 2 = 1</math> hundred.</p>	$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \\ \underline{-2} \\ 0 \end{array}$ <p>Multiply <math>1 \times 2 = 2</math>, write that 2 under the two, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 18 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \end{array}$ <p>Next, drop down the 7 of the tens next to the zero.</p>
Divide.	Multiply & subtract.	Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \end{array}$ <p>Divide 2 into 7. Place 3 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 1 \end{array}$ <p>Multiply <math>3 \times 2 = 6</math>, write that 6 under the 7, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the 1 leftover ten.</p>
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>Multiply <math>9 \times 2 = 18</math>, write that 18 under the 18, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>There are no more digits to drop down. The quotient is 139.</p>



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